An Introduction to Uncertainty, Probability and Decision Making

Prof Tim Bedford
University of Strathclyde

Prof Roger Cooke
Resources for the Future and University of Strathclyde

University of Strathclyde Business School

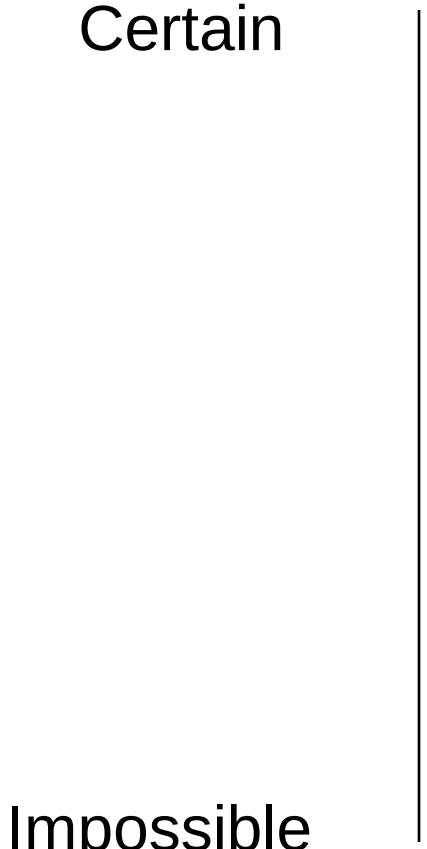
Objectives

- Understand the concept of probability as measure of uncertainty
- Connection to real world through Operational Definitions
- Understand standard rules and definitions used with probability
- Appreciate distinct roles of expert, analyst and decision maker

Probability affects us all

- “Buy your lottery tickets on a Saturday!”. A Newspaper columnist recently reported that if you buy one ticket for the national lottery on Monday, the chance of you dying before the draw the following Saturday is 2500 times the chance of winning the jackpot.
- Does this mean you should wait until Saturday to buy your ticket?

Nature of Probability



Chance of a fatality somewhere in the world in the next 24 hours?

Coin lands on head?

What is chance it rains in Glasgow tomorrow?

Rules of probability

- For any event $A \subseteq \Omega, P(A) \geq 0$
- If the events $A, B \ldots$ are exclusive then :

$$P(A \cup B) = P(A \text{ or } B) = P(A) + P(B)$$

- The certain event Ω has $\mathrm{P}(\Omega) = 1$

Two events, A, B are *exclusive* if they cannot happen together, $A \cap B = \emptyset$

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Approaches to Probability

- Combinatorial (eg Laplace)
  - based on symmetries of a problem
- Frequency-based (eg Von Mises)
  - Defined as relative number of occurrences of event of interest, within a class of infinite sequences
- Subjective (eg Savage)
  - Individual assessment measured through expressions of rational preference

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Comparison of approaches

- The probability of getting heads when you toss a coin is 0.5
  - The coin is completely symmetrical so heads and tails are equally likely
  - The long term proportion of coin tosses giving heads converges to 0.5
  - A bet where I lose 1p if heads, and win 1p if tails, is as attractive as one where I lose 1p if tails, and win 1p if heads

How does probability connect to the real world?

- *Operational Definition* required

- Notion emerged from the philosophy of science, semantic analysis: Mach, Hertz, Einstein, Bohr

> *"An operational definition is a procedure agreed upon for translation of a concept into measurement of some kind."* *Demming*

- Without knowing how to operationalise probability, it is a purely theoretical concept. Once operationalised we understand what it actually means in the real world, and what its limitations are

Are these meaningful statements?

- It will rain in Cincinnati on Jan 1 2050.
- The number of atoms in the universe is 10^{80}.
- Bread, sugar, milk, orange juice, coffee.
- Colorless green ideas sleep furiously.
- God exists.
- Next weekend I will clean my cellar.
- My flashlight has failure rate 0.001 per hour.

Truth conditions must be clear and operational for a sentence to be capable of being judged true or false.

Assigning probabilities

- Consider a statement with clear truth conditions ….
- You may be uncertain about whether this is true or not, and your subjective uncertainty (degree of belief) can be measured – in principle - through your preferences between “lottery comparisons”

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Measuring probability

- Consider
  - A bet where I lose 1p if heads, and win 1p if tails,
  - A bet where I lose 1p if tails, and win 1p if heads

- Savage's theory says that preferences are represented by a subjective probability and a utility function on outcomes, in the sense that uncertain outcomes are ranked the same by preference and by expected utility

$$P(H)U(-1) + P(T)U(1) = P(H)U(1) + P(T)U(-1)$$
$$P(T) = {}^{1}/_{2}$$

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Measuring subjective probability

- Consider two lotteries:
  - receive $ 300 if the Dow Jones goes down tomorrow, receive $ 0 otherwise
  - receive tomorrow $ 100.
- If indifferent and utility of money linear then P(Dow down)=1/3

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Limits to subjective probability

- Consider two lotteries:
  - Receive $ 1,000,000 if you clean your cellar next weekend, otherwise $ 0.
  - Receive $ 1,000,000 if the Dow Jones is lower at the end of the week, $ 0
- If prefer first then P(clean cellar)> P(Dow Jones is lower)

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Clean cellar or Dow…2

- Consider two lotteries:
  - Receive $ 1 if you clean your cellar next weekend, otherwise $ 0.
  - Receive $ 1 if the Dow Jones is lower at the end of the week, $ 0
- If prefer second then P(clean cellar)< P(Dow Jones is lower)

The point is…

- Subjective probability can be operationalised
- But
  - The measurement tool doesn't
    everything (eg uncertainty abou
    actions)
  - Is based on a theory for rational p
  - Its subjective and can be different
    rational actors

My probability of my actions is a meaningless concept

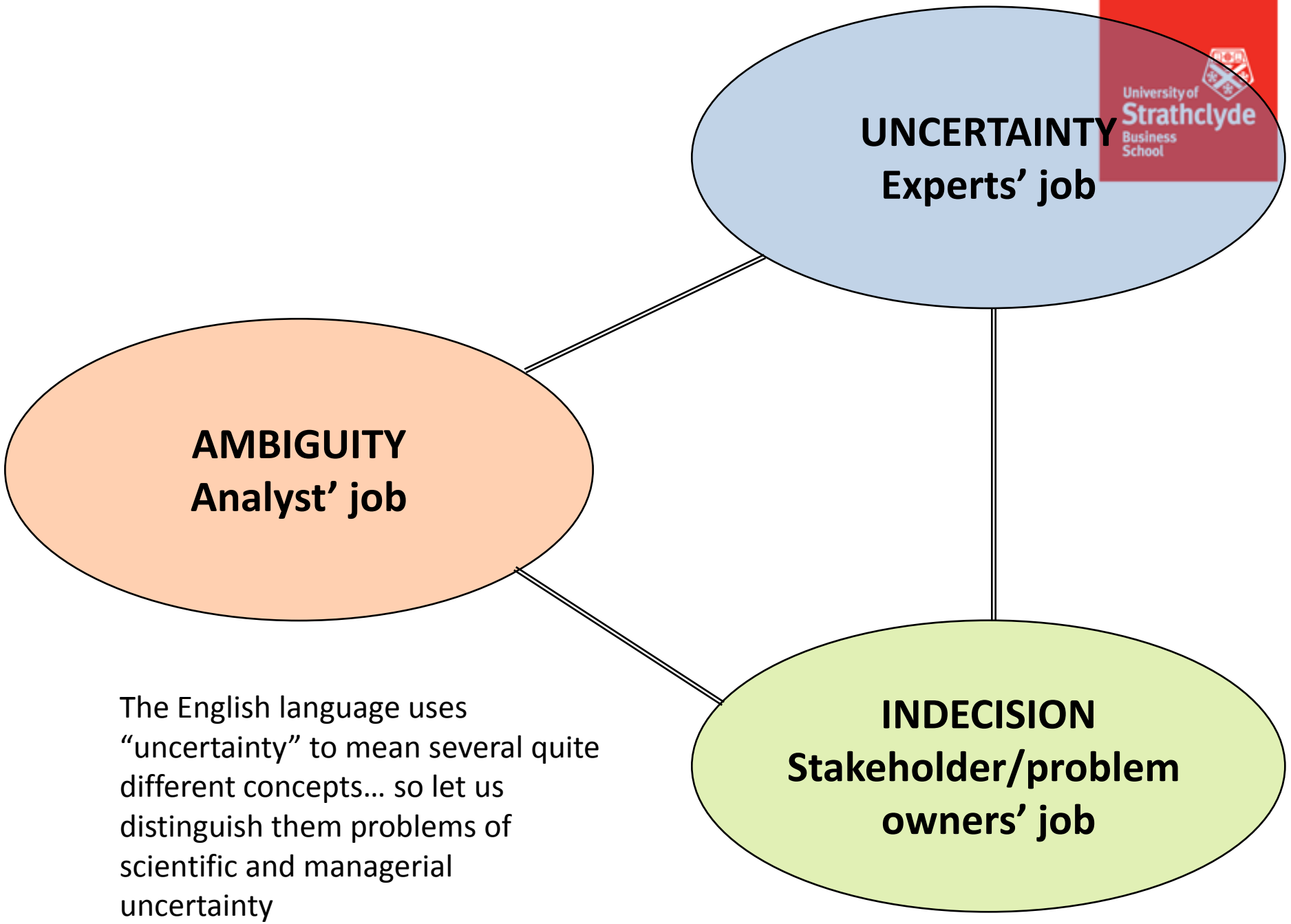
Not everyone has rational preferences, but it's

This is a challenge, so lets work on it this week!

Advantages of the Subjective definition

- Enables us to make sense of probabilities of one-off events
- Is intrinsically linked to “rational theory of decision making”, which distinguishes probabilities (uncertainty) from utilities (preference)
- Highlights that we have to make judgements about uncertainty….
- It is possible to be uncertain about a frequency (eg rate of failures), but uncertainty about uncertainty, or uncertainty about uncertainty about uncertainty (etc) doesn’t have an operational interpretation.
- Uncertainty is intrinsic to how we view the world, and can be measured through preference. It is not of itself part of the “real physical world”

- Quite magically, the notion of uncertainty we get through looking at rational preferences actually satisfies Komogorov’s Axioms of probability



The English language uses "uncertainty" to mean several quite different concepts… so let us distinguish them problems of scientific and managerial uncertainty

Back to Operational Definitions

- Key role for the analyst is to get rid of ambiguity …
- Experts may disagree because they understand the questions differently, or as yet inadequately - Experts are humans, not oracles
- We need “good enough” understanding of meaning for the purposes of the study… be pragmatic while grounded in theory!

Expert judgement

- What is it?
  - The use of structured or unstructured inputs from different individuals who have specialist knowledge of a particular domain.
- Where is it used?
  - Problem structuring
  - Model bounding
  - Model structuring
  - Model quantification
- In SEJ, the decision maker is seeking to adopt uncertainties provided by experts….But
  - Each expert will probably think differently .. May need to combine different assessments in a reasonable way
  - Some experts will be better at assessing uncertainty than others, and this is testable
- So we seek a “rational consensus” about the uncertainty

Experts' Subjective Probability: is that science?

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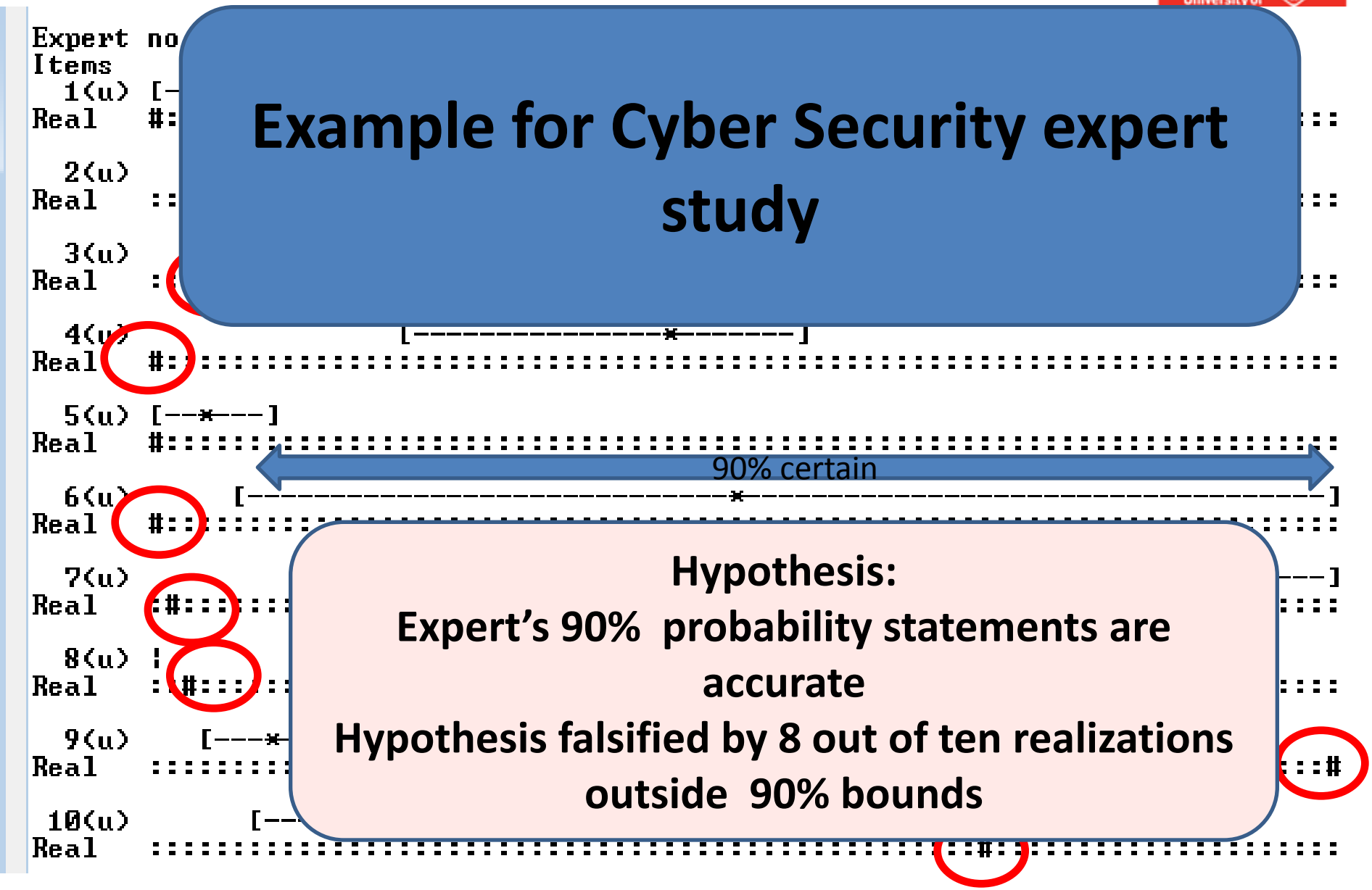
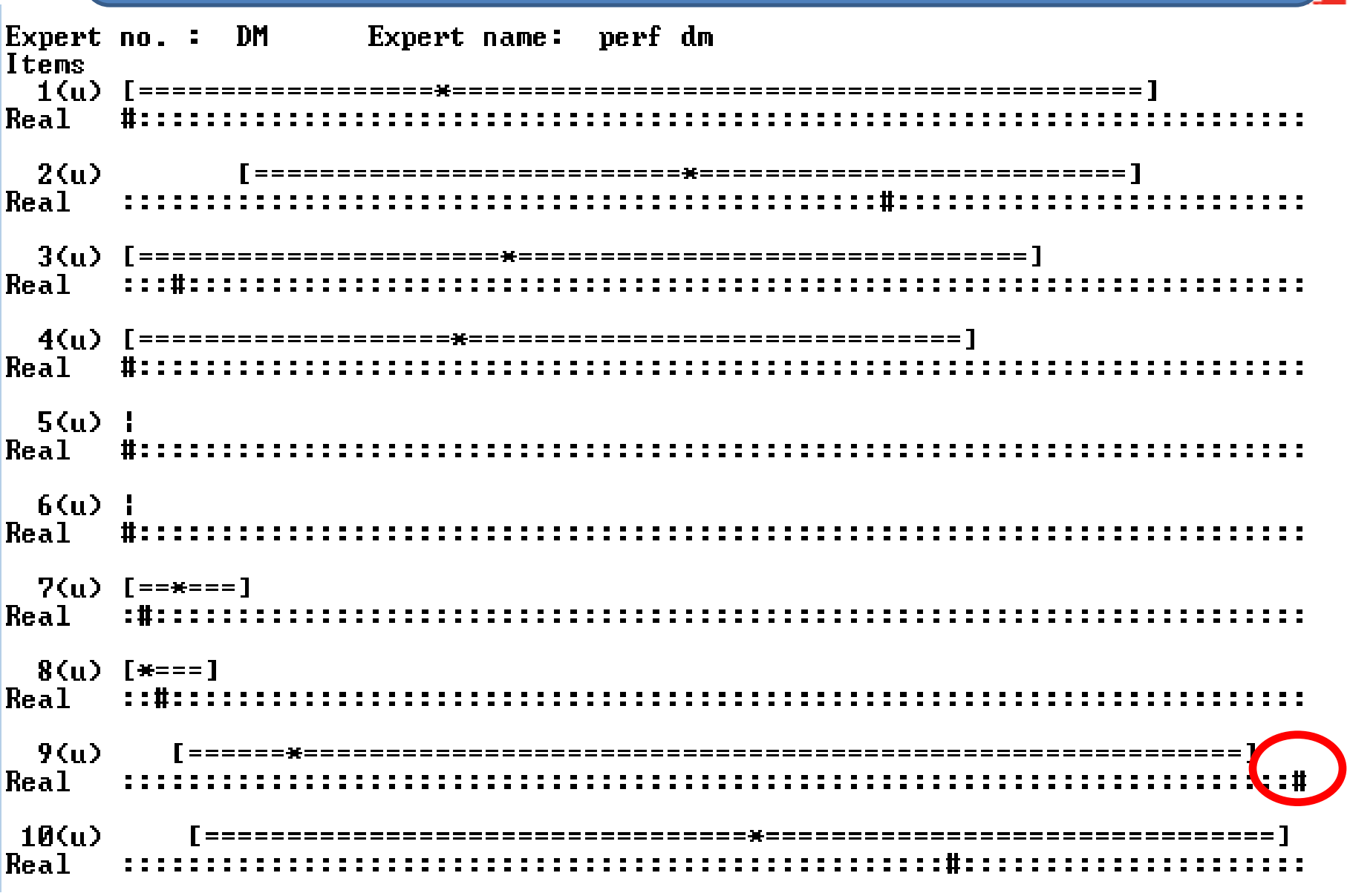
" These cancer guidelines are flexible enough to accommodate the use of expert elicitation to characterize cancer risks, as a complement to the methods presented in the cancer guidelines. According to NRC (NRC, 2002), the rigorous use of expert elicitation for the analyses of risks is considered to be quality science." (p. 3-31)

US Environmental Protection Agency, (2005), Guidelines for Carcinogen Risk Assessment, EPA/630/P-03/001F, Risk Assessment Forum, Washington, DC (March); http://cfpub.epa.gov/ncea/cfm/recordisplay.cfm?deid=116283.

Karl Popper: “is EJ falsifiable?”



Cyber Security; Performance-based Decision Maker



Current flavor of the month……

DEEP UNCERTAINTY

"In fact, the climate change debate is characterized by deep uncertainty, which results from factors such as lack of information, disagreement about what is known or even knowable, linguistic imprecision, statistical variation, measurement error, approximation, subjective judgment, and disagreement about structural models, among others (see Moss and Schneider, 2000)." U.S. Senate Committee on Commerce, Science and Transportation Hearing on "The Case for Climate

- Is the problem that it *cannot* be quantified with probability, or that it is difficult to establish a rational consensus between experts?
- Important because this is used by some as a way of diverting energy into "alternative" representations which have no operational definition, and by others to avoiding debate about appropriate actions

To sum up

- Uncertainty is represented by subjective probability
- Need to focus on clear operational definitions and reduce ambiguity
- Decision makers need support from analysts to eliminate ambiguity, and from experts to adopt consensus distributions
- Consensus distributions can be made more credible by testing expert capability

Over 100 professional applications

Partial List: Louis Goossens, Willy Aspinall, Mark Burgman, Tim Bedford, Anca Hanea, Abby Colson, Dorota Kurowicka, Oswaldo Morales, Marion Whitmann, John Rothlisberger, Bernd Kraan, John Evans, Juoni Tuomisto, Margret Palmer, David Lodge, Karen Slijkhjiis, Martijn Frijters, Tom Mazzuchi, Eric Jager, Fred Harper, Jan van Noortwijk, Nicole van Elst, Bram Meima, Arno Willems

Nuclear	EU, USNRC
Aerospace	ESTEC, NASA
Chemical Process	VROM
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Environmental Transport	EU, USNRC, VROM
Banking / Investment	SHELL, AMS Optie
Volcanoes	UK, EU
Aeronautics	VROM, AIRBUS, BA
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Ice Sheets	RL Foundation, UK
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