Structuring Probabilisitic Judgments for Integrating Decision Support Systems Group Inference, Coherence and Causality

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The Contents of this talk

EPSRC supported development of **methodologies** for probabilistic decision support for huge systems like those needed for **Food Security**.

- Outline **basic challenges** associated to Bayesian DS for huge systems.
- Links to collective decision making and **Common Knowledge** bases.
- **Delegation, aggregation, calculation** & challenges.

This work is informed through over 25 years experience in developing several platforms for Bayesian decision support

1. Begun with Simon French through the development of **nuclear decision support** tool (RODOS)
2. Continuing in ongoing work developing **food poverty** decision support tool for Local Councils.

Peculiarities of large system Bayesian decision support

- Rational **decision centres** not individuals. **Mutual Judgments of groups.**
- Decision Support $\Rightarrow$ **Utility function:** focuses on **several attributes** = **critical features**.

Example

Nuclear (Health,Public Acceptability,Cost)

Example

Food Poverty (Health, Education, Social Unrest, Cost)

- Need **quick** expected utility **scores of each policy** wrt a shared probability measure
- **Defensibility**: need narrative to justify choices post hoc.
- Challenge: Is this a formal & practical possibility in a multiagent/ dynamic environment?

Integrating the System together Probabilistically

Solution

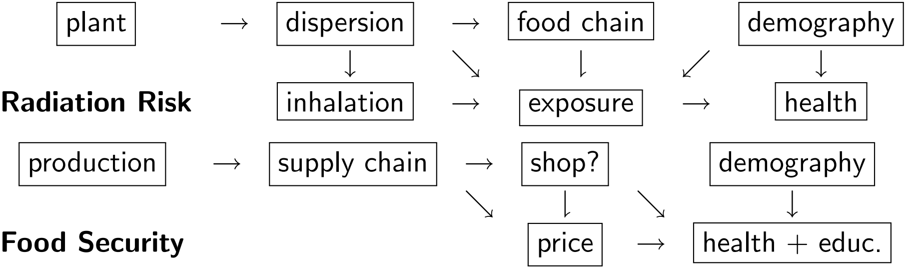
What is needed is an integrating decision support system (IDSS)!

1. Need an agreed **overarching common knowledge (CK) structure** usually consisting of:
   a. shared **qualitative** judgments about **process dependencies**, BN, tree but often customized to application
   b. an appropriate class of user **utility** function,
   c. a class of **policy choices** usually in terms of dynamic decision rules
   d. decision about **which panels to convene** & how to constitute these.
2. **Panels** of experts then populate **quantitative local domain knowledge.**
   a. sometimes supported by **prob. models**: e.g. DBNs, MDMs, trees.
   b. each donates set of **expert judgments** about prearranged conditional expectations
   c. **update** judgments dynamically as learn about domain and current process.

Necessary features of this integration

1. Need **distributivity** - problem structured so that:
    a. it is formally justified & feasible for each panel to deliver its belief judgments **autonomously.**
    b. there is **no double counting**: one panel delivers appropriate domain information to IDSS.
    c. each panel can explain their delivered expert judgments with a **supporting narrative.**
2. A **sufficiently rich** ("adequate") qualitative CK structure to provide formulae & algorithms **to knit together** panel quantitative donations to score options **correctly & quickly**.
3. Whole system must be **transparent** & **make sense** to user - smoothly embed supporting narrative.

Our 2 Running Examples: (CK on process)



Problem

Often no generic framework like OOBN appropriate: need to be customised for faithfulness and efficiency.

Solution

Develop customised semantics that still respect desirable inferential properties enjoyed by more established methods - especially distributivity.

Expressing the problem more formally.

- A **rational** expected utility maximising *SupraBayesian (SB)* takes agreed structural framework + conditional prob. models from m panels of experts.
- The **agreed CK framework** defines composition of expert panels $\{G_1, G_2, \ldots, G_m\}$. G_i delivers expert belief summaries $\{\Pi_i(d) : d \in D\}$, $i = 1.2, \ldots, m$ for each policy $d \in D$.
- Each domains has **varying complexity** & quality of information.
- Panels assumed to reason **probabilistically**: possibly supported by hybrids of trees, DBNs (ecology, nuclear), MDMs & emulators.
- SB uses $\{\Pi_i(d) : d \in D\}$ to construct conditional expectations $\Pi = f(\Pi_1, \Pi_2, \ldots, \Pi_m)$ needed to calculate her expected utilities $\overline{U}(d)$ for each $d \in D$. These are then owned by everyone.

Is this formally right? Semigraphoids & Relevance.

Let $I_0(d)$ be information *common knowledge* to all panels, $I_{ij}(d)$ be information panel i brings to $\boldsymbol{\theta}_j$ $i, j = 1, 2, \ldots, m$,
$I^+(d) \triangleq \{I_{ij}(d) : 1 \leq i, j \leq m\}, I(d) \triangleq \{I_{jj}(d) : 1 \leq j \leq m\}$

Definitions

An IDSS is *adequate* if SB can calculate $\overline{U}(d)$ from delivered outputs, *delegable* if for any $d \in D$ $\exists$ a consensus that $\boldsymbol{\theta} \amalg I^+(d) | I_0(d), I(d)$, & *separately informed* if $\amalg_{j=1}^{m} (\boldsymbol{\theta}_j, I_{jj}(d)) | I_0(d)$.

Definition

An IDSS is *sound* if adequate & by adopting the structural consensus all panel members can faithfully adopt $\overline{U}(d) : d \in D$ calculated from probs donated by relevant panels of domain experts as their own.

Theorem (Smith et al 2015)

An adequate,delegable & separately informed IDSS is sound.

There are many situations when this works!

1. **Dynamic causal frameworks** especially conducive as overarching framework.
2. **Assumptions** needed to formally justify IDSS **not automatic!** But can always check!
3. **Analogues** apply to any inferential system where scores can be defined unambiguously - e.g. belief functions.
4. Panels often need to deliver only a **few conditional expectations** (not *full* distributions). Leonelli and Smith (2015) prove **tower rule expansions** for wide classes of overarching structures where SB can instantaneously calculate all scores using prob propagation type algorithms on delivered conditional moments.
5. Need to define **customised recurrences** for each **CK structure**.
6. Policy **scores refined** as each panel **updates** its inputs in the light of new **information**.

Separable likelihoods & distributivity in a prob. IDSS

Panels often able to **update their beliefs autonomously**:

Definitions

Prior panel independence if $\amalg_{j=1}^{m} \boldsymbol{\theta}_j, |I_0(d)$. Data $\mathbf{x}$ with likelihood $l(\boldsymbol{\theta}|\mathbf{x},d)$,$d \in D$, is *panel separable* over $\boldsymbol{\theta}_i$, $i = 1, \ldots, m$ when

$$l(\boldsymbol{\theta}|\mathbf{x},d) = \prod_{i=1}^{m} l_i(\boldsymbol{\theta}_i|\mathbf{t}_i(\mathbf{x}), d)$$

where $l_i(\boldsymbol{\theta}_i|\mathbf{t}_i(\mathbf{x}))$ is fn. of $\boldsymbol{\theta}$ only through $\boldsymbol{\theta}_i$ and $\mathbf{t}_i(\mathbf{x})$ is a function of the data $\mathbf{x}$, $i = 1, 2, 3, \ldots, m$, for each $d \in D$.

Theorem

If all information conditional on the common knowledge $I_0(d)$ is data giving rise to panel separable likelihoods then prior panel independence then implies IDSS always separately informed & delegatable.

Separable Likelihoods: Do these really apply??

If overarching structure chosen well likelihoods separate surprisingly often. This is especially true when all agree a common **causal structural framework**. Thus for example we have

Theorem (Smith et al 2015)

When ∃ consensus that quantitative causal structure is a (dynamic) causal BN or casual CEG or a causal MDM & an IDSS is sound at any time t: then that IDSS remains sound under a likelihood composed of ancestral sampling experiments as well as observational sampling.

In cases when all the available data is not of the right form we can either:

1. **Approximate** - examining robustness of decisions to approximation, or
2. Apply **admissibility protocol** only admitting information into IDDS if it remains consensual and sound. e.g. allow in only **types** of sample surveys, observational studies experiment, etc.

Example: Tower rule on two panels' beliefs: toy example

CK class Panel independence between education & nutrition panel + for School hrs. lost Y by children. X nutritional balance + utility of form ensuring $\overline{U}(d)$ always expressible as fn. of $d, m_Y \triangleq E(Y), \sigma_Y^2 \triangleq Var(Y)$.
Panel 1 (nutrition) donates $\Pi_1 \triangleq \left\{ m_X \triangleq E(X), \sigma_X^2 \triangleq Var(X) \right\}$
Panel 2 (education) calculates $\left\{ \mu \triangleq E(\theta), \sigma^2 \triangleq Var(\theta), \tau^2 \triangleq Var(\varepsilon) \right\}$ & uses Bayes linear model $Y|X, \theta = \theta X + \varepsilon$ so can donate fns

$$\Pi_2 \triangleq \left\{ E(Y|X) = \mu X, E(Y^2|X) = \left(\sigma^2 + \mu^2\right) X^2 + \tau^2 \right\}$$

. Using Tower rule SB can now calculate

$$\begin{aligned} m_Y &= \mu m_x \\ \sigma_Y^2 &= \left(\sigma^2 m_x^2 + \tau^2\right) + \sigma_X^2 \left(\mu^2 + \sigma^2\right) \end{aligned}$$

Note Simple pre-determined arithmetic scores different policies. For $d \in D, \overline{U}(d)$ polynomial in contributed hyperparameters.

The combination of panels' donations in general IDSS

These type of formulae **scales up** for large dynamic probabilistic structures having a wide variety of dependence relationships: see Leonelli & Smith (2015)

- Under appropriate conditions theorems for **high dim. tensor** algebraic recurrences apply for wide variety of models.
- In **dynamic settings** typically for each t each panel donates a finite number of conditional moments for each candidate policy.
- Donations depend on **topology of overarching structure** & **form of utilities**.

Conclusions & Future Research

- Benchmark *subjectivity* - the best we can do with unambiguous information - often more helpful than "*objectivity*" for Bayesian Decision Support: directing science.
- Conditional independence & *probability* provides *agreeable combination algorithms* under CK hypotheses.
- *Often only small dim inputs* needed - (but raw BLEs -Goldstein and Wooff 2007 often not quite enough).
- Consensual *structural & causal* hypotheses often at the heart of such an IDSS! Is this therefore how Bayesians should view causality?
- New issues: *soundness* & data *admissibility*! When can IDSS demonstrate this (at least approximately)? Panels composed appropriately, right quality of information.
- Once theoretical structure clear quality of necessary *approximations* can be investigated.

Thank you Thank you Thank you

THANK YOU FOR YOUR ATTENTION!!!

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Dynamics in large system decision support

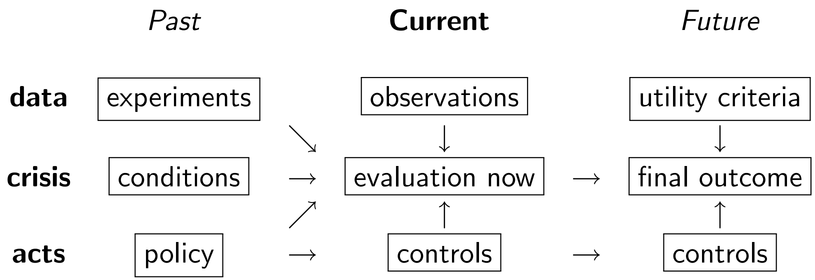
Systems often **dynamic** because:

- Short term development of **unfolding crises:** through **massive observational time series**.
- Potential and diverse effect of **enacted controls**
- Medium term changes in imperatives and horizons - in **utilities** - & in understanding of **structure of the problem**: changes in the overarching qualitative framework.
- Gradual aggregation of **experimental evidence and contextual knowledge**: new **experimental data & changing environments.**

Problem

Without overarching framework, single comprehensive probabilistic model infeasible: defensibility no single owner of judgments redundancy even if build system it quickly becomes out of date because of the necessary dynamic changes

Schemata depicting a generic dynamic framework



- "Evaluation now" extremely **complex** involving **diverse domain panel.**
- But if inferences customised to the support needed, then solutions exist that are both **formal** Smith et al (2015) and **feasible** (Leonelli and Smith, 2015)!

Examples of different structures & their Panel Independence

- BNs: Panel independence $\backsim$ global independence when considering only observational data.
- Context specific or OOBNs. Single panel responsible for shared cpts.
- Chain graphs: One panel for each variable box conditional on parents.
- MDM structures (Queen & Smith, 1993, Leonelli & Smith 2014a,2015): Panels donate distributions on dynamic regression states.
- CEG. Smith(2010) SB believes panels probs for their parts of tree independent.
- Most importantly hybrids of all these systems also often have required properties.

Examples of Unambiguous Priors.

Example

Forensic event tree. Panels allocated provision of distributions on uncertain edge probs out of particular situations in tree.

Example

Causal DBNs / MDMs. Single panels give beliefs on conditional probability table of allocated node in graph conditional on parents.

Example

Undirected graph & panels deliver a clique probability table. Not necessarily consistent since the distribution on probabilities in shared separator margins may not agree.

Beliefs and Facts: What goes into/is excluded from a system?

- Shared *beliefs* collective agrees reflect best (generally acceptable) available judgments about the global domain. Examples ci / causal/ functional relationships hardwired into system.
- Accepted *facts* Published data from well conducted experiments and sample surveys/events.

BUT most analyses implicitly or explicitly exclude certain *data*
Typical selection criteria in other contexts:

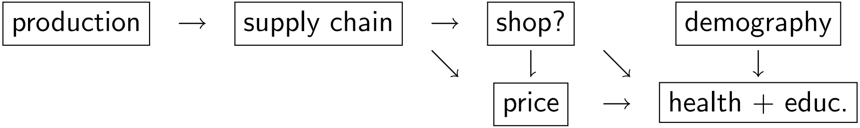
- *Compellingness* of the evidence (e.g.to user ÷ auditor/Cochraine).*Defensibility* of assumptions, *Wealth* of less ambiguous/less costly evidence.

SB updates *only* in the light of *admitted* experiments/surveys/observational studies. Cannot necessarily use *all* relevant information.

Food Security Needs: How support is delivered now.

- *Current* situation/past situations presented to Government/ Local Government through graphs & maps.
- But *no annotated predictions* of impact on poor of future events.
- Or *impact of central government changes* in legislation or evaluations of *effectiveness of different implementations* of changes.

Plan Use their standard gui & Bayesian methods to support them.



Combining Expert judgments.

- (Nuclear experiences) Modules individual expert systems
- Panel delivers inputs needed by next panel.

Panels for accident

Demography

Plant panel → Dispersal panel → Health panel

Weather panel ↗ ↘ Food, livesock panel

These panels:

1. Agree a common structure - e.g. what might influence what
2. Deliver initial vector of predictions based on their best science - then modify judgments as the crisis unfolds.

External Bayesianity

External Bayesianity (EB) if all *individually* update priors using experiment (common knowledge) - giving likelihood $l(\boldsymbol{\theta}|\mathbf{x})$ - this same as if all first combined beliefs into single panel density to accommodate their new information and then updated.
EB property characterises the *logarithmic pool*

$$\overline{\pi}(\boldsymbol{\theta}|\mathbf{w}) \propto \prod_{i=1}^{k} \pi_i^{w_i}(\boldsymbol{\theta})$$

where $\mathbf{w} = (w_1, \ldots, w_k)$ weights, reflecting credibility of different experts, sum to unity.
Collective appears Bayesian from outside irrespective of sampling and order of information. Consistent with the Strong Likelihood Principle. Preserves integrity of panel independence over time.

Recapping the Problem

- Collective agrees set of qualitative (e.g. conditional independence) assumptions about $\{Y_i : 1 \leq i \leq n\}$ conditional on $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots \boldsymbol{\theta}_m)$ whatever $d \in D$.
- Let $\Pi = f(\Pi_1, \Pi_2, \ldots, \Pi_m)$ be the distributional statements about $\boldsymbol{\theta}$ available to the user. Panel beliefs $\{\Pi_j(d) : 1 \leq j \leq m, d \in D\}$ the *only* quantitative inputs to the collective beliefs $\Pi(d)$ about $\boldsymbol{\theta}$.

Note: not trivial that $\Pi(d)$ is function of $\Pi_j(d) : 1 \leq j \leq m$.
e.g distribution of parameters of $\mathbf{Y} = (Y_1, Y_2)$ is not fully recoverable from the two marginal densities $\pi_i(\boldsymbol{\theta}_i)$,provided by G_i, $i = 1, 2$ e.g. no covariance between Y_1 and Y_2 .

Example: decision support after a nuclear accident

Many panels of experts/statistical models in the system:

- Power station described by a Bayesian Network - **Panel** nuclear physicists, engineers and managers.
- Accidental release into the atmosphere or water supply the dangerous radiation will be distributed into the environment, **Panel** atmospheric physicists, hydrologist, local weather forecasters....
- Taking outputs of dispersion models and data on demography and implemented countermeasures predict exposure of humans animal and plants of the contaminant. **Panel** biologists Food scientists, local adminstrators, ..
- Taking outputs giving type and extent of exposure predict health consequences: **Panel** epidemiologists, medics, genetic researchers
- And so on ...

Big demands of the 21st Century more generally

- Complex *domain specific probabilistic expert systems* inform different parts of process.
- Cannot single probabilistic *composite*: too big! ever changing modules, only interested in certain outputs of these modules.
- So *Integrating Decision Support System (IDSS)* essential: pasting together the pertinent outputs of autonomous dynamic expert judgments to deliver benchmark numerical evaluation (with justification) of each candidate policy.
- Panels deliver *updated* judgments autonomously as a function of much in depth analysis.

Coherence in a decision centre over many systems

Principle 1

- An IDSS should be *coherent*.
- Coherence requires virtual responsible *SupraBayesian(SB)* to represent the centre.
- IDSS evaluate SB's *expected utility function*, for candidate unfoldings and policies.

Note Bet caller - regulators, stakeholders, users, other experts *actually there* to test out integrity."*coherent*" = no-one without domain knowledge can exploit SB's implied preferences over specific types of gambles.

Faithfulness for a decision centre over many systems

Principle 2 An IDSS should be *faithful:*

- IDSS to express broad qualitative consensus over qualitative features of problem.
- SB's *single probability* distribution over space needed to calculate expected utilities. *"best" most consensual/faithful/defensible probabilities*: (e.g. Smets,2005).
- SB should adopt beliefs of *relevant panel* of domain experts (coded with probs). IDSS *justifiable*: relevant domain experts to field regulator queries about faithfulness/plausibility.

Principle 3 An IDSS must be *feasible*, *transparent* & *fast*.

Example: Observables a pair of binary variables

- $\mathbf{R} = \mathbf{Y} \triangleq (Y_1, Y_2)$. Panel G_1 inputs about $\theta_1 \triangleq P(Y_1 = 1)$.
- Panel G_2, $\theta_{2,0} \triangleq P(Y_2 = 1 | Y_1 = 0)$ and $\theta_{2,1} \triangleq P(Y_2 = 0 | Y_1 = 1)$.
- Distribution of $\mathbf{R}$, $\overline{\boldsymbol{\theta}} \triangleq \left(\overline{\theta}_{00}, \overline{\theta}_{01}, \overline{\theta}_{10}, \overline{\theta}_{11}\right)$ given by the polynomials

$$\begin{aligned}\overline{\theta}_{00} &= (1-\theta_1)(1-\theta_{2,0}), \overline{\theta}_{01} = (1-\theta_1)\theta_{2,0}, \\ \overline{\theta}_{10} &= \theta_1(1-\theta_{2,1}), \overline{\theta}_{11} = \theta_1\theta_{2,1}\end{aligned}$$

- G_1 donates densities $\Pi_1 = \{\pi_1(\theta_1, d) : d \in D\}$.
- G_2 gives densities $\Pi_2 = \{(\pi_2(\theta_{2,0}, d), \pi_2(\theta_{2,1}, d)) : d \in D\}$.

Example: The Queen in Danger!!

Example

Panel G_1 domain is margin of binary Y_1 - $\theta_1 = P(Y_1 = 1)$ (Y_1 queen comes in contact with a particular virus). Panel G_2 domain margin of binary Y_2, $\theta_2 = P(Y_2 = 1)$. (Y_2 when queen exposed suffers an adverse reaction).G_1 says $\theta_1 \backsim Be(\alpha_1, \beta_1)$ and G_2 says $\theta_2 \backsim Be(\alpha_2, \beta_2)$. No decision will affect these distributions. Agreed structural information is $Y_1 \amalg Y_2 | (\theta_1, \theta_2)$,

Case1: User has a separable utility

$$u_1(y_1, y_2, d_1, d_2) = a + b_1(d_1)y_1 + b_2(d_2)y_2$$

G_i needs only supply $\mu_i \triangleq \mathbb{E}(\theta_i) = \alpha_i(\alpha_i + \beta_i)^{-1}$, $i = 1, 2$. No need to be concerned about dependency.

Example

Case 2

- Interest is only in $W \triangleq Y_1 Y_2$ (whether queen is infected). So

$$u_2(w, d_{12}) = a + b_{12}(d_{12})w$$

  where $\mathbb{E}(W) = \mathbb{E}(\theta_1\theta_2)$.
- If collective assumes *global independence* $\Rightarrow$ distribution $\theta_1\theta_2$ is well defined.
- Then $\mathbb{E}(\theta_1\theta_2) = \mu_1\mu_2$ - so G_i needs only supply μ_i, $i = 1, 2$.
- However Global independence not *only* choice!

An Alternative Prior

Suppose $\alpha_1 + \beta_1 = \alpha_2 + \beta_2 \triangleq \sigma$. Panels donate (μ_1, μ_2, σ), where $\sigma = \gamma_{00} + \gamma_{10} + \gamma_{10} + \gamma_{11}$, $\pi \backsim Di(\gamma_{00}, \gamma_{10}, \gamma_{01}, \gamma_{11})$,

$$\begin{aligned} \alpha_1 &= \gamma_{10} + \gamma_{11}, \beta_1 = \gamma_{00} + \gamma_{01} \\ \alpha_2 &= \gamma_{01} + \gamma_{11}, \beta_2 = \gamma_{00} + \gamma_{10} \end{aligned}$$

- This collective prior consistent with panel margins but *not* global independence.
- Collective parameters $(\mu_1, \mu_2, \sigma, \rho)$, $\rho \triangleq \sigma^{-2} (\gamma_{11}\gamma_{00} - \gamma_{10}\gamma_{011})$
- Collective's $\mathbb{E}(\theta_1\theta_2) = \gamma_{11}\sigma^{-1} = \mu_1\mu_2 + \rho \neq \mu_1\mu_2$ unless $\rho = 0$.
- So $\mathbb{E}(\theta_1\theta_2)$ is not identified from inputs.

Now assume global independence

- Panels supplement judgments by independently randomly sampling.
- Collective needs only two updated posterior means $\mu_i^*.i = 1, 2$.
- So all data of this form allows distributed inference.

Problem 1: Global independence critical for distributivity. Even in Case 1 when only individuals margins of θ_1, θ_2 needed if collective did not believe $\theta_1 \amalg \theta_2$ it learns about θ_2 - through G_2's experiments will modify distribution of θ_1.

Problem 2 :Even if global independence is justified, assuming experiments of two panels never mutually informative also critical.

Example of data set: table of counts below (Case 2)

$Y_1 \backslash Y_2$	0	1		
0	5	45	50	$n - x_1$
1	45	5	50	x_1
	50	50	100	
	$n - x_2$	x_2		

- Each panel updates using only their respective margin (with weak priors) $\Rightarrow \mu_i^* \simeq 0.5$, $i = 1, 2 \Rightarrow \mathbb{E}(\theta_1 \theta_2)$ to be approximately 0.25.
- OTOH with whole info $\mathbb{E}(\theta_1 \theta_2) \simeq 0.05$.i.e. five times smaller!

(Note structural independence assumption: $Y_2 \amalg Y_1 | (\theta_1, \theta_2)$ looks dubious)

Non-compatible sampling (either case)

Binomial sample 100 units like queen, *acquiring* disease, so prob $\phi \triangleq P(W = 1)$. See 5 infected.

- In either case collective easily incorporates this information directly: e.g. giving ϕ a beta prior and treating data as random sample. However, without further assumptions such data impossible for G_i to *individually* update $\pi_i(\theta_i)$.
- Ignore this information $\div$ uniform priors $\Rightarrow$ vastly overestimate the probability.
- So $\pi(\theta_1\theta_2)$ no longer decomposes into a G_1 density and a G_2 density: Sampling induces dependence.

So problems quite involved! Distributed panels need to reflect form of typical input data as well as areas of expertise.