Some problems in uncertainty modelling and foundational issues (in relation with IS1304 EJNET)

David Rios

Glasgow, April 2014

Some caveats

- My biases (but I tried to open my mind)
- My limits (but I have read a lot)
- Focusing on modeling and foundational issues
- Many, inevitably, at the interface with several other WGs
- Not all at same detail level (again my bias)
- Indeed, most of them rather informal questions

$ECUATE_j$

1. Eliciting EJ
2. Combining EJ
3. Using EJ
4. Assessing EJ
5. Technology and EJ

6. A testbed project

1. Elicitation for main distributions

- O'Hagan et al (2006) compare 10+ methods for eliciting the parameters of a Beta(a,b) distribution (conjugate of binomial) trying to come out with a best method
- Similar studies for other conjugate distributions so as to get a best practice catalogue
  - Observables (predictive)
  - Quantiles
  - Probabilistic Inversion method
- NB: Multivariate distributions. WG2

1. Preference modelling

- Farquhar (1984)
- Draw new light on such methods?
- Distribution of preference over stakeholders.
  - Ranges of reactions/Sensitivity analysis
- Weighted additive utility… Multiplicative utility
- Error models for utilities

1. Deep uncertainties, long term uncertainties

- Meaningless?
- Worse performance at deep, long term tasks
- Decision under risk vs Decision under uncertainty….. Knight etc…
- Strategy in Stewart, French, Rios (not me!!!)

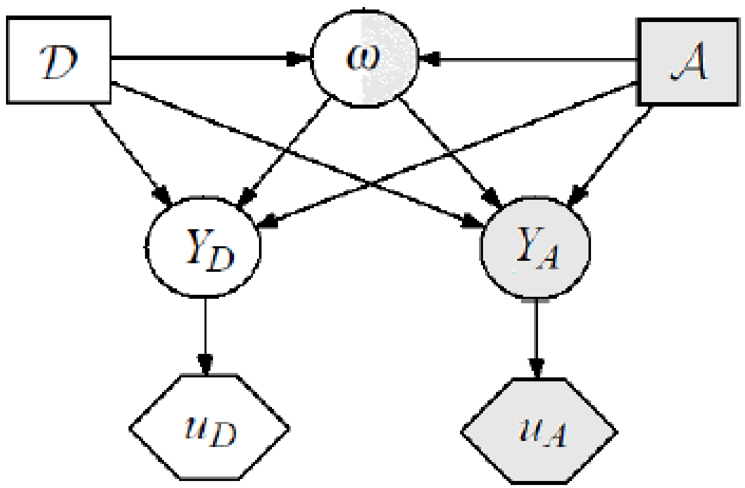
1. Adversarial uncertainty modelling

- RA enhanced to include adversaries ready to increase our risks

- S-11, M-11 lead to large security investments globally
- Many modelling efforts to efficiently allocate such resources
- Parnell et al (2008) NAS review
  - Standard reliability/risk approaches not take into acocunt intentionality
  - Game theoretic approaches. Common knowledge assumption…
  - Decision analytic approaches. Forecasting the adversary action…
- Merrick, Parnell (2011) review approaches commenting favourably on Adversarial Risk Analysis

Basic Problem

$\{d_1, \ldots, d_m\}$



$\{a_1, \ldots, a_n\}$

$$\psi_D(d,a) = \int u_D(a,d,\omega) p_D(\omega|a,d) d\omega$$

$$\psi_A(d,a) = \int u_A(a,d,\omega) p_A(\omega|a,d) d\omega$$

	a
d	$(\psi_D(d,a), \psi_A(d,a))$

$$\max_d \psi_D(d) = \sum_{i=1}^{n} \psi_D(d, a_i) p_D(a_i) = \sum_{i=1}^{n} \left[\int u_D(a_i, d, \omega) p_D(\omega|a_i, d) d\omega \right] p_D(a_i)$$

$p_D(a)$

Non strategic opponent. I

- A lacks memory. Dirichlet-multinomial model

$$(p_1, \ldots, p_n) \sim \mathcal{D}(\alpha_1, ..., \alpha_n)$$

$$(p_1, \ldots, p_n)|\text{data} \sim \mathcal{D}(\alpha_1 + h_1, ..., \alpha_n + h_n)$$

$$p_D^{NS}(a_i) = E(p_i|\text{data}) = \frac{\alpha_i + h_i}{\sum_{j=1}^{n}(\alpha_j + h_j)}, i = 1, \ldots, n.$$

$$\max_d \sum_{i=1}^{n} \psi_D(d, a_i) p_D^{NS}(a_i)$$

Non strategic opponent. II

- A remembers his last attack, her last defense and the results. Matrix-beta prior model

$$(p_1, \ldots, p_n)|a_i, d_j, \omega \sim \mathcal{D}(\alpha_1^{ij\omega}, ..., \alpha_n^{ij\omega})$$

$$(p_1, \ldots, p_n)|a_i, d_j, \omega, \text{data} \sim \mathcal{D}(\alpha_1^{ij\omega} + n_1^{ij\omega}, ..., \alpha_n^{ij\omega} + n_n^{ij\omega})$$

- To control size growth, mixture model

$$p_D(a|a_i, d_j, \omega) = w_1 p_D(a|a_i) + w_2 p_D(a|d_j) + w_3 p_D(a|\omega).$$

Inference and forecast through a Gibbs sampler

Ficticious play

Level-k thinking opponent I

- D needs to solve

$$d^* = \arg\max_d \left[\sum_a \psi_D(d,a) p_D(a)\right]$$

- For this, she thinks about A's problem

$$\begin{aligned} a^* &= \arg\max_a \left[\sum_d \psi_D(d,a) p_A(d)\right] \\ &= \arg\max_a \left[\sum_d \int u_A(d,a,\omega) p_A(\omega|a,d) d\omega\right] p_A(d) \end{aligned}$$

- She does not know $(u_A, p_A(.|.), p_A)$
- Models uncertainty through $(U_A, P_A(.|.), P_A)$

$$A|D \sim \arg\max_a \sum_d \left[\int U_A(d,a,\omega) P_A(\omega|a,d) d\omega\right] P_A(d) \qquad p_D(a) = p_{A|D}(a)$$

Simulate

Level-k thinking opponent II

$$(U_A, P_A(\cdot\,|\,\cdot), P_A)$$

$$D|A^1 \sim \arg\max_d \sum_a \left[\int U_D(d,a,\omega) P_D(\omega|a,d) d\omega \right] P_D(a),$$

Repeat from $i=1$

Find $P_{D^{i-1}}(A^i)$ by solving

$$A^i \mid D^i \sim \arg\max_{a\in\mathcal{A}} \sum_{d\in\mathcal{D}} \left[\int U_A^i(a,d,\omega) P_A^i(\omega\,|\,a,d) d\omega \right] P_A^i(D^i = d)$$

with $(U_A^i, P_A^i(\cdot\,|\,\cdot), P_A^i) \sim F^i$

Find $P_A^i(D^i)$ by solving

$$D^i \mid A^{i+1} \sim \arg\max_{d\in\mathcal{D}} \sum_{a\in\mathcal{A}} \left[\int U_D^i(a,d,\omega) P_D^i(\omega\,|\,a,d) d\omega \right] P_D^i(A^{i+1} = a)$$

with $(U_D^i, P_D^i(\cdot\,|\,\cdots), P_D^i) \sim G^i$

$i = i+1$

MacLay, Rothschild, Guikema (2012)   Rios, DRI (2012)

Prospect opponent

- EU model OK for D (as giving prescriptive advice)
- EU model OK for A???
- Terrorist psychology and logistics suggest optimising terrorists (cutthroat capitalism)

$$\arg\max_{a} \left[\sum_{d} \int v_A(d,a,\omega) w_A^1(p_A(\omega|a,d)) d\omega \right] w_A^2(p_A(d))$$

$$A|D = \arg\max_{a} \left[\sum_{d} \int V_A(d,a,\omega) W_A^1 P_A(\omega \,|\, a,d)) d\omega \right] W_A^2(P_A(d))$$

Reconciling and learning about opponent model

- Use a mixture of opponent models

$$p_D(a) = \sum_{i=1}^{k} q_i p_D^i(a)$$

- Model averaging to optimize

$$\max_d \sum_a \psi_D(d,a) \left(\sum_{i=1}^{k} q_i p_D^i(a) \right) = \max_d \sum_{i=1}^{k} q_i \left[\sum_a \psi_D(d,a) p_D^i(a) \right]$$

- Model selection to learn about weights.

1. Adversarial uncertainty modeling

- Additional operational principles
- More complex structures
- Is it worth going up one level in the hierarchy
  - More accurate, but more work
  - Value of information gained
- Multiple experts stopping at different levels
- ….

1. Multivariate extreme models

- Many extreme problems are multivariate
  - E.g. in extreme weather, floods+droughts (possibly because of El Niño-La Niña effects)
- Univariate extreme models relatively well understood
  - Choice of thresholds?
  - Mixture models
- Need to model dependence

2. Aggregating rules

- New aggregating rules still appearing
  - Hora et al (2013) Median aggregation
  - Lichtendahl et al (2013) Averaging quantiles
  - Jose et al (2013) Trimmed av quantiles
- A comparison with gold standards required
- Modelling as a mixture problem (prior on weights to model dependence)

3. Risk Matrix methods

OCCURRENCE CATEGORY / EVENT TYPE					
RISK MATRIX	Without Safety Effect	Significant Incident	Major Incident	Serious Incident	Accident
Extremely Unlikely					
Extremely Remote					
Remote					
Reasonably Possible					
Frequent					

ARMS, Bowtie, IRP,…

3. Risk Matrix Methods

- ICAO, for civil aviation
- COSO, for auditing
- MAGERIT, HMG Std 1, for IT Security
- IPCC SREX, for extreme weather risks
- ….

3. Risk Matrix Methods

- From Cox (2008)
  - Ambiguous inputs and outputs
  - Insufficient detail
  - Suboptimal resource allocation
  - Errors

3. Risk matrix methods

If, leaving apart laziness, we lack of resources to perform a proper risk analysis…

- How much do we lose for not doing the whole thing?
- As in ordinal data, latent variables with thresholds for likelihoods. Similarly for impacts/utilities.
- Combining expert judgements in such setting

3. Back to discretisation…

$$P(A|\hat{\theta})$$

$$P(A|data) = \int P(A|\theta)\pi(\theta|data)d\theta$$

$$P_{MC}(A|data) = \frac{1}{n}\sum_{i=1}^{n} P(A|\theta_i)$$

$$\tilde{P}(A) = \sum_{i=1}^{m} P(A|\theta_i)p_i$$

Reduced order models (Grigoriu, 2009)

Also usable in reporting (as in risk matrices)

4. Scoring rules

- Scoring rules for elicitation (Savage)

- New scoring rules appearing (eg Merkle, Steyvers, 2013)
- Compare with gold standards
- Role in elicitation

- How are they modified with extreme incentives and disincentives?

4. Sensitivity Analysis

- Parameters -----→ Inf, Pred, Risk Ass, DM

Baucells, Borgonuovo (2013), DRI, Ruggeri (2000)

- EJ--→ Parameters --→ Inf, Pred, Risk Ass, DM

5. EJ Technology

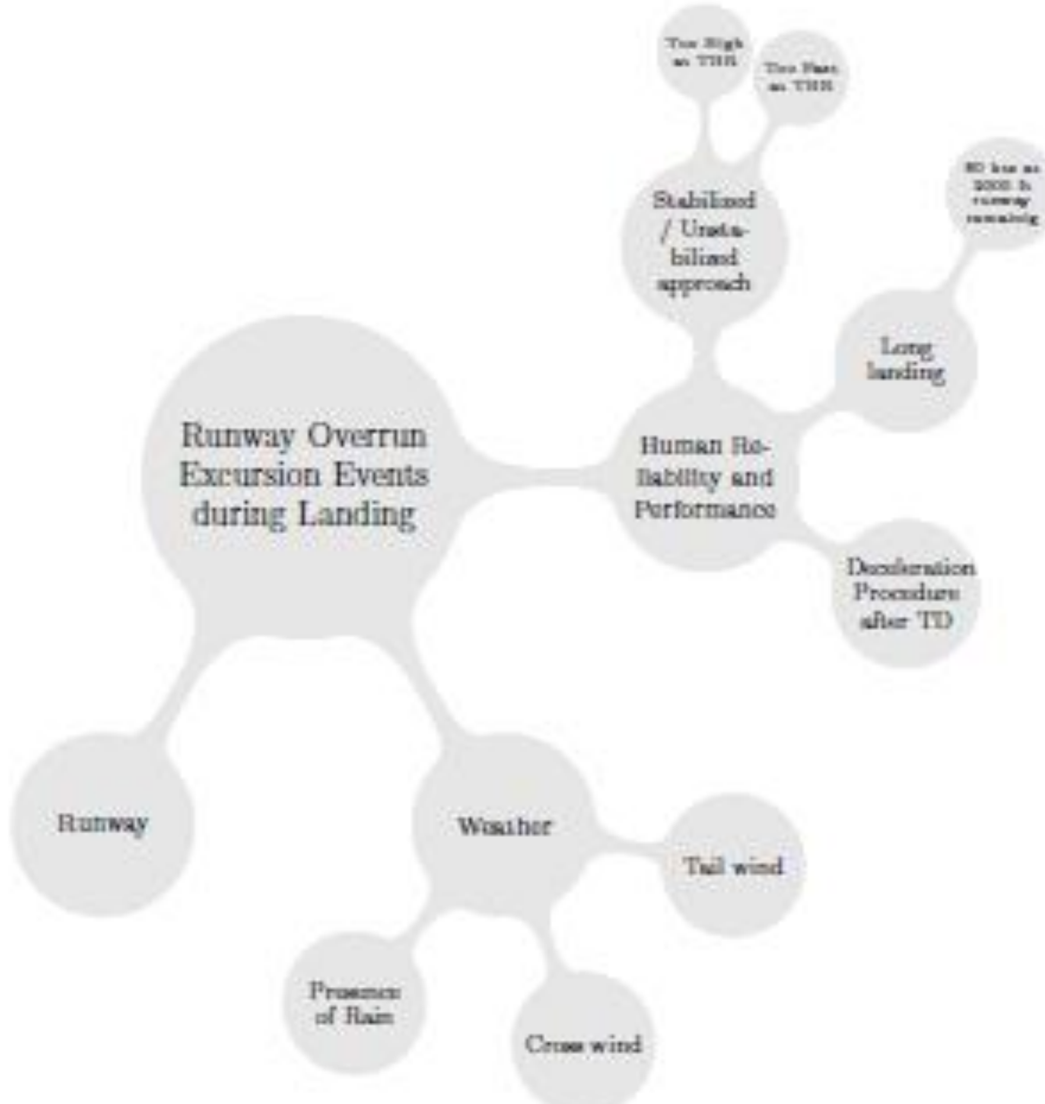
- Many of the above ideas, and others already around, and others coming from IS1304 WGs could be turn into software supporting EJ services

  - EJ Web Services
  - Links to R, Winbugs etc…
  - Open Source

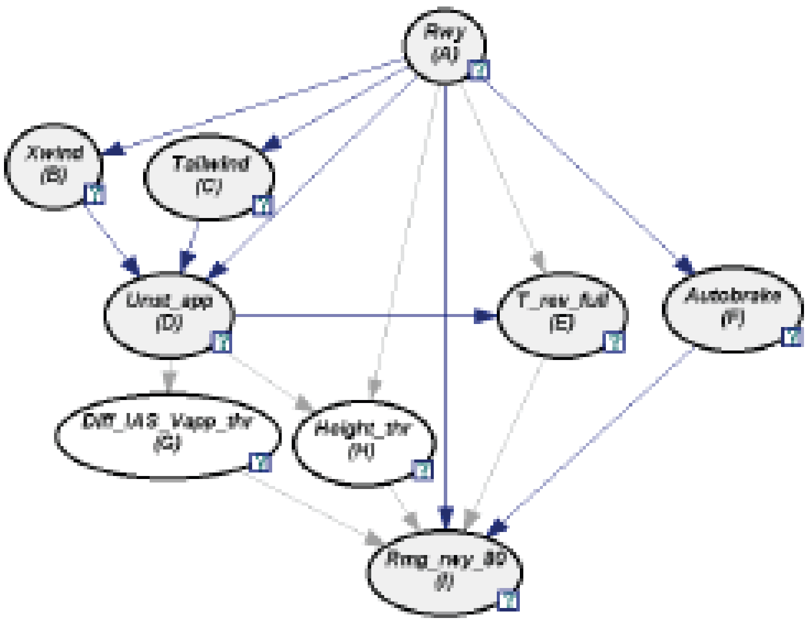
5. The limits of expert judgement?

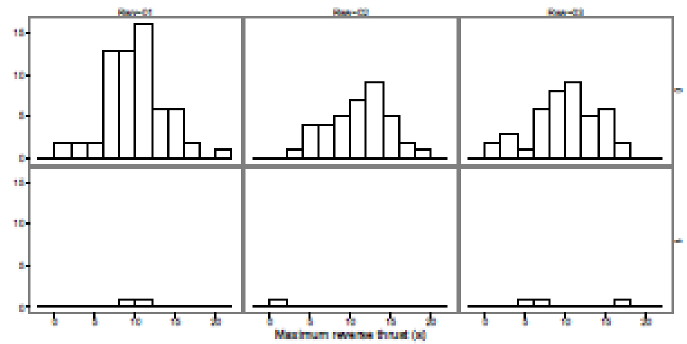
- EJ in times of Big Data?
- BD: The end of science as we know it….
- How do we combine EJ with Big Data

Runway excursions



Runway excursions





$$E_j = p_{0j} I_0 + p_{1j} E_j^{+},$$
$$p_{0j} + p_{1j} = 1,$$
$$p_{0j}, p_{1j} \geq 0,$$


(a) Runway 01 end


(b) Runway 02 end


(c) Runway 03 end

$$p(e|a_j, d=0, data) = \frac{1 + n_j - y_j}{2 + n_j} I_0 + \frac{1 + y_j}{2 + n_j} \frac{\alpha_j \beta_j^{\alpha_j}}{(\beta_j + e)^{\alpha_j + 1}}.$$

$$p(e|a, d=1, data) = \frac{\bar{\alpha}_j (\bar{\beta}_j)^{\bar{\alpha}_j}}{(\bar{\beta}_j + e)^{\bar{\alpha}_j + 1}},$$

6. A testbed project

❑ SAFETY

- Safety is Critical in Civil Aviation

We are doing well but...

...not enough

STATE SAFETY PROGRAMME?

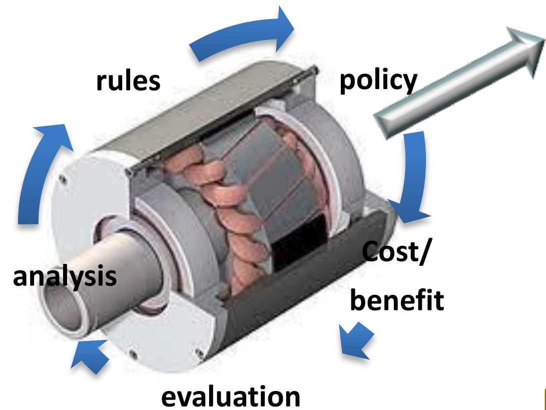
- ICAO : “An integrated set of regulations and activities established by a State aimed at managing civil aviation safety”
- Support strategic decision-making in adopting better decisions when allocating scare resources to higher safety risk areas
- To implement preventive approach for safety oversight and to manage safety at a State level, States must develop a State Safety Program (SSP)

State Safety Program

Input

Data

- ✓ Operational
- ✓ Financial
- ✓ HHRR
- ✓ Legal...



Output

Strategic decisions

Programs

- ✓Safety Action Plan
- ✓Air Operator Certificates
- ✓Licenses
- ✓Airworthiness...

PROJECT METHODOLOGY

- **Incident forecasting**
- **Incident consequence assessment and forecasting**
- **Risk mapping**
- **Deciding on interventions (resource allocation)**
- **Detailed analysis of chosen incidents**

- **Pervaded by risk matrices**
- **From reactive to predictive**
- **Expert Judgement, multiple experts (with different interests), multiattribute preference modelling, extreme event modelling, dependence, use for policy making,….**

Thanks

david.rios@urjc.es