Expert Judgement Informed Sequencing of Reliability Growth Tasks

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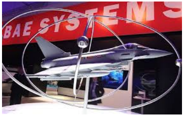
Reliability Enhancement Methodology and Modelling (REMM) project

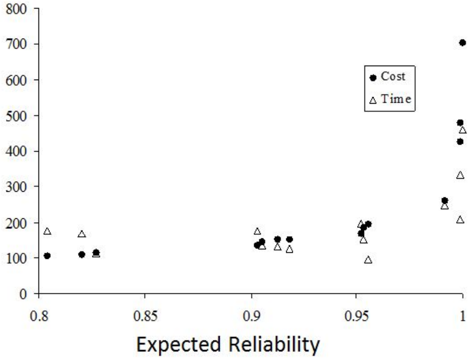


Using reliability tools to ensure greater product reliability, reduce costs of rework and spares provisioning and provide greater customer confidence and satisfaction.

- University of Strathclyde: Lesley Walls and John Quigley.
- University of Loughborough.
- Several industrial partners.
- RAF Reliability Group.
- UK Department of Trade and Insdustry

$$\text{Concern} \quad \overset{}{\begin{bmatrix} 0.5 \\ 0.4 \\ 0.2 \\ 0.5 \\ \vdots \\ 0.2 \end{bmatrix}} \quad \overset{\text{Activity}}{\begin{bmatrix} 0.9 & 0 & 0.3 & \cdots & 0.7 \\ 0.5 & 0 & 0 & \cdots & 0.9 \\ 0.7 & 0 & 0.2 & \cdots & 0.8 \\ 0.2 & 0.5 & 0.4 & \cdots & 0.7 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0.9 \end{bmatrix}}$$

- During hardware product development, engineers will regularly meet to discuss potential engineering concerns.
- An efficacy matrix can be created detailing:
  1. The probability of engineering concerns being a fault.
  2. The probability of each of the possible tasks revealing the fault.



- Cost can be optimised to achieve certain levels of reliability.
- Time is not independent of cost but is not optimised - extra cost can reduce time.
- Steep finish demonstrates cost and time to achieve very high reliability.

- During product development designs are analysed by performing reliability tasks, identifying design weaknesses.
- These weaknesses are then designed out and the system reliability improves.
- Examples include fault tree analysis, failure mode and effects analysis, accelerated life testing, etc.
- Such tasks can be resource intensive and expensive.
- Outcomes of tasks not mutually exclusive: multiple tasks expose the same weakness (some weaknesses not exposed).
- Target: a method to identify the optimal sequence of reliability tasks for a product/system.

- Let $i = 1, \ldots, I$ be engineering concerns and $j = 1, \ldots, J$ be possible reliability tasks. Then

$$X_i = \begin{cases} 1, & \text{if concern } i \text{ is realised,} \\ 0, & \text{otherwise,} \end{cases}$$

- and $\Pr(X_i = 1) = \lambda_i, \; \Pr(X_i = 0) = 1 - \lambda_i$.
- Let $p_{i,j}$: probability task j realises fault i if it exists.
- We can elicit $\lambda_i, p_{i,j}$ from engineers inside the organisation.
- If faults are independent then $R(t) = \prod_{i=1}^{I} R_i(t)^{X_i}$.
- Having performed each task $\theta_j = 1$ or not $\theta_j = 0$:

$$\mathrm{E}_D \left\{ \mathrm{E}_{X|D} \left[R(t) \right] \right\} = \prod_{i=1}^{I} \left[1 - (1 - R_i(t)) \, \lambda_i \prod_{j=1}^{J} (1 - p_{i,j})^{\theta_j} \right].$$

- We are interested in quantities of the form $\Pr(R(t) \geq R_0) = \alpha$, for some α close to one.
- First transform
$$\eta(t) = \log\left[R(t)\right],$$
and assume $\eta(t) \sim N(m(t), v(t))$.
- We are interested in $\Pr(R(t) \geq R_0) = \Pr(\eta(t) \geq \log R_0)$.
- Specify exactly in terms of $m(t), v(t)$ as
$$m(t) = \mathrm{E}_d\left\{\mathrm{E}_{X|d}\left[\eta(t)\right]\right\},$$
$$v(t) = \mathrm{E}_d\left\{\mathrm{E}_{X|d}\left[\eta(t)^2\right]\right\} - \mathrm{E}_d\left\{\mathrm{E}_{X|d}\left[\eta(t)\right]\right\}^2.$$
where $d_i = 1$ if fault i found.
- The moments are computationally expensive to calculate.

- Rare event approximation:

$$-\log\left[R(t)\right] = \sum_{i=1}^{I}(1-R_i(t))\lambda_i - \sum_{i=1}^{I}(1-R_i(t))\lambda_i \sum_{j=1}^{J}\theta_j(1-\alpha_{i,j}),$$

- where $\alpha_{i,j}$ is 1 if task j finds fault i given that fault i exists.

$$\begin{aligned} m(t) &= -\left[\sum_{i=1}^{I}(1-R_i(t))\lambda_i - \sum_{i=1}^{I}(1-R_i(t))\lambda_i \sum_{j=1}^{J}\theta_j(1-p_{i,j})\right], \\ v(t) &= \sum_{i=1}^{I}\left[(1-R_i(t))\lambda_i\right]^2 \sum_{j=1}^{J}\theta_j(1-p_{i,j})p_{i,j}. \end{aligned}$$

- Number of computations for a sequence of length J;
  - Exact method: $N_1 = 2^{2I+1} \times {}^J P_J$.
  - Rare event approximation: $N_2 = 2IJ \times {}^J P_J$.
- If $I = 5, J = 5$ then $N_1 = 245,760$ and $N_2 = 6000$ and if $I = 15, J = 14$ then $N_1 = 1.87 \times 10^{20}$ and $N_2 = 3.66 \times 10^{13}$.

- Suppose Y_j is the cost and χ_j is the time on test of task j.
- Optimal sequence of tasks:

$$\max_{s \in S} \left[\mathrm{E}_d \left\{ \mathrm{E}_{X|d} \left[U(Y, \chi) \right] \right\} \right],$$

for total cost Y and time on test χ, where $U(Y, \chi)$ is the utility function.

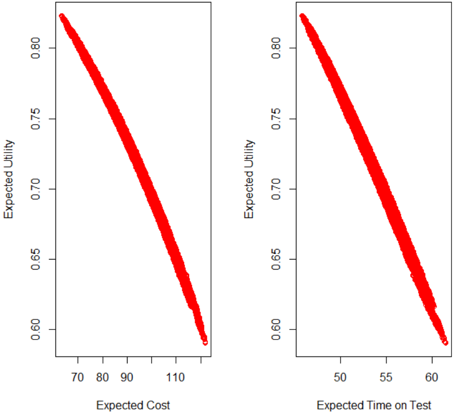
- Y, χ depend on when we reach our target.
- If cost and time on test are utility independent then:

$$U(Y, \chi) = p_1 U(Y) + p_2 U(\chi) + p_3 U(Y) U(\chi).$$

- Assuming the decision maker is risk averse:

$$U(Y) = 1 - (Y/Y_0)^2, U(\chi) = 1 - (\chi/\chi_0)^2.$$

- Suppose that engineers identify 15 possible design flaws in a product and there are 9 possible tasks to identify these flaws.
- This means that in all there are 362,880 possible sequences of the tasks which could be carried out.
- Each task has a cost of between 0 and 50 units and duration of 0 to 20 units.
- The target reliability is 0.8, the maximum time on test is 150 units and the maximum total cost is 258 units.
- Assume a constant failure rate for each fault type (can relax this).
- λ_i between 0 and 0.5, 54% of the $p_{i,j}$ are equal to 0 and the rest are between 0 and 0.5 and μ_i is 0.02.



Task	4	2	8	1	9	5	7	6	3
Cost	17	52	63	63	109	114	135	138	165
Time	9	10	18	24	40	43	48	52	67
Probability	0.0002	0.10	0.46	0.82	0.97	0.99	1.00	1.00	1.00

- We want to approximate expected utility $U(\cdot)$ with a surrogate $f(\cdot)$ such that, for sequences z_k,

$$f(z_k) > f(z_{k'}) \text{ if } U(z_k) > U(z_{k'}).$$

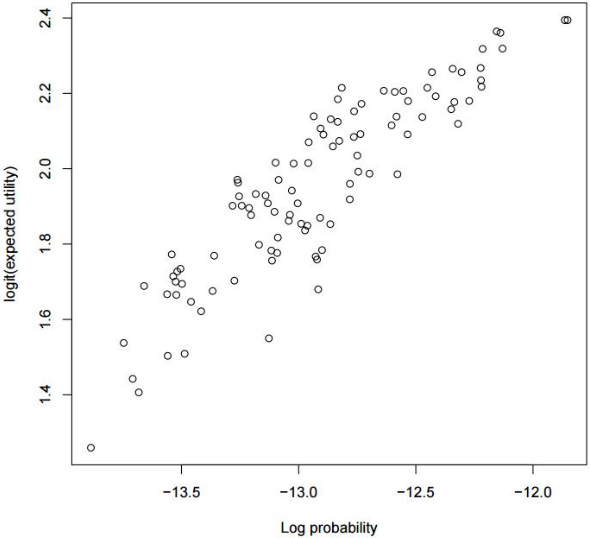
- We would like $f(\cdot)$ to be faster to evaluate than $U(\cdot)$.
- Suppose that for task j we have γ_j which is proportional to the probability that task j is scheduled first.
- Then (Plackett-Luce), the probability of a sequence $z = (z_1, \ldots, z_J)$ is

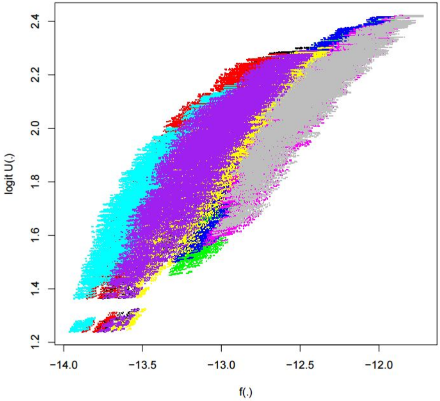
$$\Pr(z \mid \gamma) = \prod_{j=1}^{J} \frac{\gamma_{z_j}}{\sum_{m=j}^{J} \gamma_{z_m}}.$$

- We want $\gamma = (\gamma_1, \ldots, \gamma_J)$ to maximise $\mathrm{Corr}(\beta, l^{\gamma})$, where

$$\beta = \mathrm{logit}(U(z)), l_k^{\gamma} = \log(\Pr(z_k \mid \gamma)).$$

- We choose $f(\cdot)$ to be $\log \Pr(z_J, \ldots, z_1 \mid \gamma)$.


logit(expected utility)
2.4
2.2
2.0
1.8
1.6
1.4
−13.5
−13.0
−12.5
−12.0
Log probability



- In this case, the sequence with the highest $f(\cdot)$ does not have the highest expected utility.
- The sequence with the highest expected utility is the 9th highest in $f(\cdot)$.

- We have presented a model for system reliability which was developed to mirror the engineering process.
- To sequence reliability tasks, we can use expected utility, but the resulting solution becomes computationally intractable.
- To overcome this, we can find an approximate optimal allocation using the rare event approximation and surrogate functions.
- The utility functions can be used to express risk aversion in decision makers.
- Future work is to investigate the properties of the surrogate function.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HE CALLED SHOTGUN...
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