

## Fat Tails and Tail Dependence

Roughly speaking, fat tailed distributions are those with polynomial decay. They arise frequently in the study of extremes and in the study of loss distributions. The Pareto distribution  $P(X > x) = x^{-\alpha}$ ,  $x > 1$ ,  $\alpha > 0$ , is a limiting form of many distributions in this class. Depending on the fatness of tail, these distributions can frustrate standard statistical tools and play havoc with our intuitions. Among the non-intuitive features of fat tailed distributions are:

- Infinite mean or infinite variance.
- Independent sums do not converge to a normal distribution with shrinking variance, rather they converge to a distribution with the same tail index as the summands.
- Historical averages do not converge but whiplash.
- The probability of a next worst case being at least twice as severe as the current worst case does not decrease as the number of observations increases.
- Conditional value at risk (CV@R) is infinite.
- Correlations, regression coefficients are unstable

Very little work has been done on eliciting information on tails of a distribution from experts, and this could be a high value research target for many fields.

Related to fat tails and less well understood is the phenomenon of tail dependence. This refers to the tendency of extreme values in one variable to occur together with extreme values in another variable. It is trivial that a weak global positive correlation across a set of variables will be amplified as the variables are summed. Far from trivial is the question when summing variables will induce and amplify tail dependence. Tail dependence is obviously of critical importance for risk transfer, securitization and risk management, should be of concern for the banking investment and insurance sectors. Data for assessing tail dependence is often difficult to acquire, as it requires multivariate extremes. Can experts be enlisted to quantify tail dependence in the absence of quantitative data? This question deserves all the research we can spare.

### **Fat-Tailed Distributions: Data, Diagnostics, and Dependence**

Roger M. Cooke, Daan Nieboer, Jolanta Misiewicz  
RFF Discussion Paper 11-19-REV | September 2011

### **The Unholy Trinity: Fat Tails, Tail Dependence, and Micro-Correlations**

Carolyn Kousky, Roger M. Cooke  
RFF Discussion Paper 09-36-REV | November 2009

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